Mean Flow Streamlines of a Finite-Bladed Propeller

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Introduction

THE increase in the development of V/STOL aircraft and high-speed ships in recent years has refocused attention on the theory, design, and performance of propellers. Examination of the associated problems that have arisen reveals that a principal need is the refinement of the general mean flowfield streamlines.

The purpose of this note is to present the results of calculations of the mean flow streamlines for two distributions of propeller bound-blade circulation. One is a constant corresponding to the conventional actuator disk model; the other is a simple analytic form chosen to approximate closely the familiar Goldstein optimum. The calculations are based upon earlier studies[1,2] of the propeller-induced velocity field.

It is hoped that these results will facilitate the renewed efforts in propeller research and development by imparting a stronger physical feeling for the flow.

Formulation

Consider the uniform motion at zero incidence of a lightly loaded propeller of blade number N and circulation distribution Γ in an inviscid, incompressible freestream of speed U. The blades, of radius R_p, are rotating about the propeller centerline at a constant angular velocity Ω. A cylindrical, propeller-fixed coordinate system (x,r,θ) is chosen such that the propeller disk is normal to the x axis and is located at $x = 0$.

The classical vortex lifting-line model is used to represent the propeller and its wake. With this model, the axial, radial, and tangential induced velocities at any field point can be determined straightforwardly from the Biot-Savart law. The time-average, or steady, velocity components are determined by Fourier-analyzing these induced velocities to find the zeroth harmonic.[1] Knowledge of the steady axial velocity enables us then to compute the mass flow through any cross section. Since, by definition, there is no mass flow across any streamtube of the flowfield, we can use lines of constant mass flow, in turn, to calculate the mean flow streamlines.

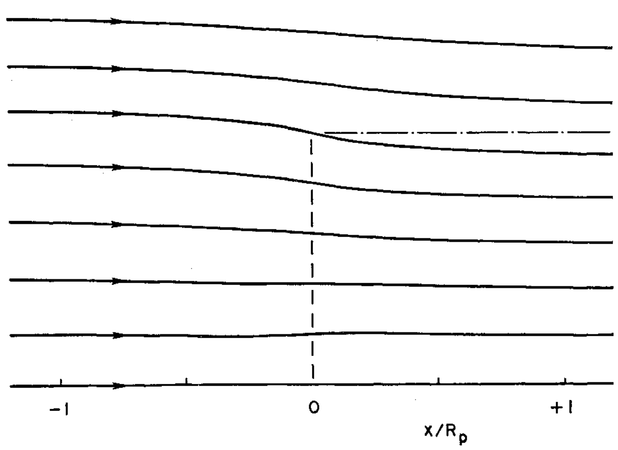

Fig. 1 Mean flow streamlines for $C_T = 1.0$ and representative blade circulation. Propeller disk indicated by (- -); $r = R_p$ by (—·—).

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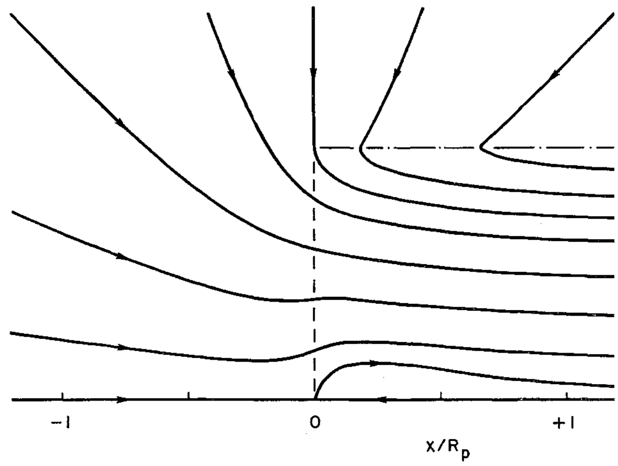

Fig. 2 Perturbation mean flow streamlines for representative blade circulation.

The mass flow per unit time M through a cross section of radius $\mathbf{r}$ is

$$M(x,\mathbf{r}) = 2\pi\rho \int_0^{\mathbf{r}} r\,u(x,r;\Gamma)dr \tag{1}$$

where ρ is the fluid density and u is the axial velocity, which can be written as

$$u = U + UC_T\mathbf{u}(x,r;\Gamma) \tag{2}$$

Here, $UC_T\mathbf{u}$ is the steady axial perturbation velocity due to the propeller which has been evaluated and tabulated in Ref. 1. The propeller thrust coefficient $C_T \equiv T/\frac{1}{2}\rho U^2\pi R_p{}^2$ is given by

$$C_T = \frac{2\Omega N}{\pi U^2 R_p{}^2}\int_0^{R_p} \xi\Gamma(\xi)d\xi \tag{3}$$

If we nondimensionalize M by $\rho U\pi R_p{}^2$, or the freestream mass flow rate through the propeller disk, and all lengths by R_p, then substitution of Eq. (2) into Eq. (1) gives

$$\bar{M} = \bar{\mathbf{r}}^2 + 2C_T\int_0^{\bar{\mathbf{r}}} \bar{r}\mathbf{u}(\bar{x},\bar{r};\Gamma)d\bar{r} \tag{4}$$

where the bar denotes a dimensionless quantity.

The integration over $\bar{r}$ is most easily carried out by means of a "running integral"[3] using the trapezoidal rule. This enables us, for a given $\bar{x}$, to compute $\bar{M}$ for a series of equal increments in $\bar{\mathbf{r}}$. Constant values of $\bar{M}$, and so the mean flow streamlines, follow by interpolation in the results.

Results

Figure 1 shows the streamline pattern with $C_T = 1.0$ for the representative blade circulation distribution $\Gamma \propto \bar{r}(1 - \bar{r})^{1/2}$, which approximates the well-known Goldstein optimum. The streamline passing through the propeller tip contracts far downstream to 91% of the propeller radius, or equivalently, to 83% of the corresponding value far upstream. There is virtually no difference between these results and those calculated for the case of constant blade circulation. We could show the behavior of the streamlines at other representative thrust coefficients but the over-all features do not differ a great deal from those of Fig. 1.

It is interesting to consider the streamlines corresponding to the perturbation field alone of the propeller; i.e., only the contribution of the integral term on the right-hand side of Eq. (4) to $\bar{M}$. We might regard this perturbation contribution as a crude "first approximation" to the flowfield which occurs in the static operating condition.

The perturbation streamline pattern is given in Fig. 2 for the representative distribution of bound-blade circulation discussed before. Several new features appear. First, we

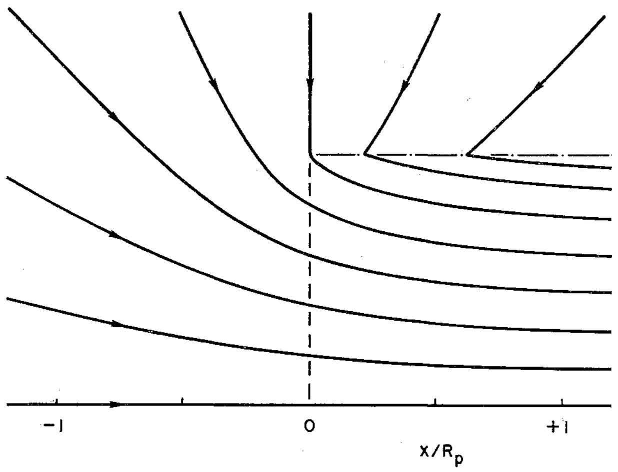

Fig. 3 Perturbation mean flow streamlines for constant blade circulation.

find that for radii greater than R_p, the streamlines are exactly symmetric about the propeller plane. By continuity, this substantiates the doubling of the slipstream velocity at infinity as found from a momentum balance. The streamline passing through the propeller tip approaches with an infinite slope and then contracts far downstream to a radius of 70.7% of the propeller radius, as can be easily verified from the continuity equation. Downstream of the propeller plane and outside the tip streamline, the flow approaches the propeller and then abruptly is swept back downstream as it first crosses the vortex system trailing from the blades. Perhaps the most striking feature that we notice, though, is the appearance of a "natural hub" created within the flowfield. This is caused by the local inner trailing vortex system. These vortices are of opposite sign to those further outboard and so induce a flow "upstream." As a result, we expect that the hub for a real propeller probably does not disturb the flow as much as might be thought beforehand. In Fig. 1, this natural hub effect has almost been masked out by the freestream contribution, which dominates the flowfield.

Figure 3 shows the results for the perturbation streamlines for the case of constant circulation distribution. This can be related to the streamline pattern for a circular sink disk of uniform strength; see for example Ref. 4. Comparing the results of Fig. 3 with those of Fig. 2, we find that the extent of the influence of the nonuniformity of the circulation is very local and appears only in the immediate vicinity of the propeller itself. Again, the streamlines are exactly symmetric about the propeller plane outside the slipstream and the tip streamline contracts to a value of 70.7% of the propeller radius as before.

For the static condition, we have been able to correlate certain features of these perturbation streamlines, e.g., the rapid turning of the flow near the propeller tip and the overall slipstream contraction, with experimental smoke-visualization pictures. Other features that do not correlate serve to remind us of the basic deficiency of the classical vortex model, which assumes that the wake is made up of regular helices and does not contract.[5] Nevertheless, it is very useful at least insofar as a simple picture of the static flowfield is concerned.

Conclusions

Results have been presented which show the effect of the shape of the propeller blade circulation distribution upon the mean flow streamlines. This effect is found to be confined to the very immediate vicinity of the propeller itself, and decreases quite rapidly as the forward flight velocity increases. For the perturbation streamlines, a natural hub is observed in the flowfield for the representative circulation and so the real propeller hub probably does not disturb the flow as much as might be thought.

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Effect of Back Pressure on Nozzle Thrust

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Nomenclature

A = streamtube flow area
F = thrust
m = mass flow rate
M = Mach number
P = total pressure
p = static pressure
V = velocity
γ = ratio of specific heats

Subscripts

e = exit plane, actual conditions
i = exit plane, ideal conditions at pressure p_0
0 = ambient atmosphere
t = upstream of nozzle, stagnation conditions

It is well known that the thrust of a supersonic nozzle is a maximum when the nozzle exit static pressure p_e is equal to the ambient atmospheric pressure p_0. Experience indicates this is not true for sonic or subsonic nozzles, especially when operating at low nozzle pressure ratios. In this note we shall examine the variation of nozzle thrust when the exit static pressure differs from ambient pressure. This case has received little analytical attention in the past, although there are many practical situations in which such variations can occur. Curvature in an exhaust duct, for example, can produce pressure gradients at the nozzle exit plane,[1] which permits some of the flow to exhaust at a pressure lower than ambient. For small radii of curvature the pressure variation may be quite marked. In other cases the nozzle exit plane may be located in a pressure field that is above ambient pressure. Examples are a nozzle exhausting from a boattail body, or a nozzle discharging normal to a ground plane (ground-effect machine).

Consider a small streamtube in which the flow is approximately one-dimensional. For a constant mass flow m_e we

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